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THE TRANSVERSE RESPONSE OF SANDWICH PANELS TO AN UNDERWATER SHOCK WAVE

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The response of submerged structures to an underwater shock wave involves both structural and fluid behaviour. For sandwich structures the response to an initial shock wave in the transverse direction is significantly different from that of a homogeneous structure. This is due to the elastic properties of the core in the sandwich. For a homogeneous structure one cavitation zone is initially developed and the position of this zone is dependent on the assumed cavitation pressure. At a sandwich structure two cavitation zones initially appear, one adjacent to the structure and another away from the structure, depending again on the assumed cavitation pressure. The response of a sandwich panel in the transverse direction is also investigated, using a combination of a numerical method and finite elements, developed for the fluid–structure interaction problem. The method includes the appearance of cavitation in the fluid, it is found that, as the sandwich section is moving, the faces in the sandwich oscillate about the core. The nonlinear properties of the faces give rise to considerable transverse strain which may very well be high enough to cause delamination in the faces, debonding between the faces and the core, or core failure.

1. INTRODUCTION

WHEN IT COMES TO THE DESIGN of military naval vessels exposed to underwater explosions, the resistance against shock waves is of major concern. A shock analysis of a vessel involves several aspects, such as (a) the arrival of the initial shock wave, (b) the decay of the initial shock wave, (c) local cavitation due to the structural reflected shock wave or surface reflected shock wave, (d) fluid–structure interaction, (e) local cavitation collapse and (f) structural response. The whole sequence of events leads to a problem which is very complicated and difficult to predict. For simple geometries, closed-form solutions can be found, but for practical structures numerical methods are unavoidable. However, even the numerical solution of the coupled governing equations becomes intractable for structures with hundreds of degrees of freedom. Therefore, approximate methods have been developed to solve the fluid–structure interaction problem. These methods utilize the Plane Wave Approximation (PWA) which is an approximation for the early stages of the response and was first developed by Mindlin and Bleich (1953). The method has been used extensively by Geers (1978) for the early-time predictions of the response of surface ships and submarines exposed to underwater explosions.

For sandwich shell structures the effect of cavitation can be significantly different from that of a homogeneous shell structure. In the case of a homogenous structure, the need to consider the deformation in the thickness direction is usually neglected; however, this is not the case for a sandwich structure which usually has a soft core.

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Both Driels (1980) and Handleton (1985) have investigated this problem considering a homogenous structure, and Driels includes the effect of nonzero cavitation pressure. If the fluid is able to withstand tension pressures then a cavitation zone is opened a distance away from the structural surface and a certain amount of water vibrates with the structure and increases its mass. Hayman (1995) investigated a sandwich structure using the same approach, but included the elasticity in the core and found that two cavitation zones appeared. A cavitation zone adjacent to the structure appears first, and a short time later one develops away from the structure. The latter is similar to that shown by Driels but further away from the structure. Both Driels and Hayman use the pressure for the free field which is valid up to a time of one decay length. This means that approximations are valid up to that time; however, by using the equations of hydrodynamics we may get round this problem.

Numerical examples using a simple l-D finite-element model, for different sandwich configurations are evaluated. Expressions from Cole (1948) are used to determine the initial conditions (incident pressure) from an explosive charge. The pressure on the panel surface is affected by the local cavitation in the fluid, and the numerical examples show that the faces in a sandwich panel oscillate about the core as the whole sandwich section is moving and that the global response is similar to that of a homogeneous plate. The nonlinear properties of the faces give rise to considerable strain and stress in the transverse direction.

2. GOVERNING EQUATIONS

2.1. Structural Equations

The motion of a structure is governed by the equation

$$M\ddot{w} + C\dot{w} + Kw = p(t), \tag{1}$$

where M is the structural mass, C the structural damping, K the structural stiffness, w the structural displacement and $p(t)$ the applied time-varying load.

For a structure initially at rest we have the initial conditions

$$w(0) = \dot{w}(0) = 0. \tag{2}$$

At the structural surface we also have the compatibility condition between the velocity of the structure and the fluid particle velocity u:

$$\dot{w}(t) = -\,u(0, t). \tag{3}$$

2.2. Initial Shock Pressure

The initial shock wave (free-field pressure, p_i) can be approximated by the following equation given by Cole and is valid up to one decay length θ:

$$\begin{aligned} p_i &= p_0 \mathrm{e}^{-t/\theta}, \\ u_i &= -\,p_i/(\rho_0 c_0), \end{aligned} \tag{4}$$

which is an approximation for the early time of the initial shock wave. Here p_0 is the maximum pressure and θ the decay constant which are given by the expressions

$$p_0 = 56{\cdot}6(Q^{0{\cdot}33}/R)^{1{\cdot}15}, \text{ (MPa)},$$

$$\theta = 0{\cdot}084 \cdot Q^{0{\cdot}33}(\mathrm{Q}^{0{\cdot}33}/R)^{-0{\cdot}23}, \text{ (ms)},$$

where Q is the weight of the explosive charge in kg and R is the distance between the charge and the structure in m. These formulas are valid for HBX charges (HBX being a type of explosive).

2.3. Hydrodynamic Equations

In linearized (acoustical) formulation, the equations of hydrodynamics are

$$\frac{\partial u}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} = 0, \qquad \frac{\partial p}{\partial t} + \rho c^2 \frac{\partial u}{\partial x} = 0, \tag{5}$$

where $u(x, t)$ is the only component of velocity, $p(x, t)$ the deviation from the hydrostatic pressure, and ρ and c are respectively, the density of the medium at rest and the velocity of sound therein. In linear nonstationary problems of fluid–structure interaction the potential formulation is usually used. Such an approach is convenient because it gives a possibility to solve a boundary problem for a single (wave) equation; in particular, potential formulations were used by Galiev (1981) and Geers (1981) in their numerical models.

3. ANALYTICAL SOLUTION

Recently, Hayman has presented an analytical solution for a sandwich plate. For a sandwich plate two cavitation zones will open. First, one adjacent to the plate, due to reflection of the transmitted wave in the rear face, and a short time later a new cavitation zone, similar to Driels' solution, will open a distance away from the plate surface due to reflection of the incident wave on the front face.

The theory of the pressure field in front of a sandwich panel is summarized here. The equation of motion for the sandwich front face, which is assumed to be thin, can be written as

$$(p_i(t) + p_{r1}(t) - p_{c1}(t) = m_f \dot{v}_f(t), \tag{6}$$

where $p_i(t)$ is the free-field pressure, $p_{r1}(t)$ is the reflected pressure in the water from the face, $p_{c1}(t)$ is the transmitted pressure into the core, m_f is the mass of the front face per unit area and $\dot{v}_f(t)$ is the acceleration of the front face. For compatibility at the front face of the sandwich, the following relation between the velocities must hold:

$$u_i(t) + u_{r1}(t) = u_{c1}(t) = v_f(t), \tag{7}$$

where $u_i(t)$, $u_{r1}(t)$, $u_{c1}(t)$ are the particle velocities at the panel face for the incident, reflected and transmitted waves. The velocities are given by

$$u_i = \frac{p_i(t)}{\rho c}, \quad u_{r1} = -\frac{p_{r1}(t)}{\rho c}, \quad u_{c1} = \frac{p_{c1}(t)}{\rho_c c_c}. \tag{8}$$

Solving equations (6)–(8) and combining with the initial condition $v_f(t = 0) = 0$ we obtain the pressures for the transmitted and reflected waves which are as follows:

$$p_{c1}(t) = A(\mathrm{e}^{-\alpha t} - \mathrm{e}^{-\beta t}), \tag{9}$$

$$p_{r1}(t) = B_1 \mathrm{e}^{-\alpha t} + B_2 \mathrm{e}^{-\beta t}, \tag{10}$$

where the constants are given by

$$A = \frac{2\gamma p_0}{(\beta - \alpha)}, \quad B_1 = p_0 - B_2, \quad B_2 = \frac{2(\beta - \gamma) p_0}{(\beta - \alpha)},$$

$$\alpha = \frac{1}{\theta}, \quad \beta = \frac{(\rho c + \rho_c c_c)}{m_f}, \quad \gamma = \frac{\rho_c c_c}{m_f}.$$

When the transmitted wave $p_{c1}(t)$ has passed through the core and reaches the rear face of the sandwich, it is assumed that the entire pressure is reflected back into the core. Since this face is supported only by air this is a good approximation. The wave reaches the rear face after a time interval t_1, where

$$t_1 = d/c_c, \tag{11}$$

in which d is the core thickness and c_c is the sound velocity in the core given by the expression $c_c = \sqrt{E_c/\rho_c}$. Here E_c is Young's modulus and ρ_c the density of the core, respectively.

When this reflected wave reaches the front face again, after time $t_2 = 2t_1$, one part is then reflected back into the core and one part transmitted into the water. The pressure of the transmitted wave into the water is given by the expression

$$p_{r2}(t) = E_1 \mathrm{e}^{-\alpha t_2} + (E_2 + E_3 t_2)\mathrm{e}^{-\beta t_2} + E_4 \mathrm{e}^{-\gamma t_2}, \tag{12}$$

where the constants are given by

$$E_1 = \frac{2(\beta - \gamma) D_1}{(\beta - \alpha)}, \quad E_2 = -(E_1 + E_4), \quad E_3 = -2(\beta - \gamma) D_2, \quad E_4 = -2D_3,$$

$$D_1 = \frac{2\gamma(\alpha + \gamma) p_0}{(\beta - \alpha)(\gamma - \alpha)}, \quad D_2 = -\frac{2\gamma(\beta + \gamma) p_0}{(\beta - \alpha)(\beta - \gamma)}, \quad D_3 = \frac{4\gamma^2 p_0}{(\gamma - \alpha)(\beta - \gamma)}.$$

The total pressure in the water in front of the sandwich panel can now be calculated by superimposing the solution of equations (4), (10) and (12) and by replacing t with $(t + x/c)$:

$$p_{\mathrm{tot}}(t) = p_i(t) + p_{r1}(t) + p_{r2}(t). \tag{13}$$

The definition of x is positive from the front face and into the core, see Figure 1.

If we have a sandwich panel with 4 mm thick GRP faces and a 60 mm thick core of 200 kg/m^3 PVC, the total weight of the panel will be about 24 kg/m^2. When exposed to a shock wave with $p_0 = 3{\cdot}0$ MPa and $\alpha = 2580\ \mathrm{s}^{-1}$ at the panel surface, cavitation occurs first at the panel about 0·103 ms after the initial wave strikes the panel; see Figure 2. This is due to the arrival of the transmitted wave at the fluid-structure interface. At the same time the reflection of the incident wave, in the front face, is then progressing out in the water and cuts off the tail of the initial shock wave after about 0·123 ms at a distance of about 160 mm from the panel surface.

4. NUMERICAL METHOD

We consider a semi-infinite region $x \geq 0$, supposing that the sandwich faces are located at the coordinates $x = 0$ and $x = d$ and that they are infinitely thin, are just consisting of a mass m_f. The initial shock wave with amplitude p_0 and decay constant θ comes to the front face at the instant $t = 0$ having the direction of its propagation coinciding with the negative direction of the X-axis. The initial conditions ($t = 0$) for equation (5) are then given by equation (4).

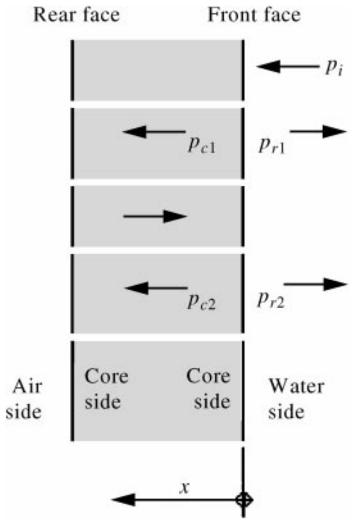

Figure 1. Waves through the sandwich plate.

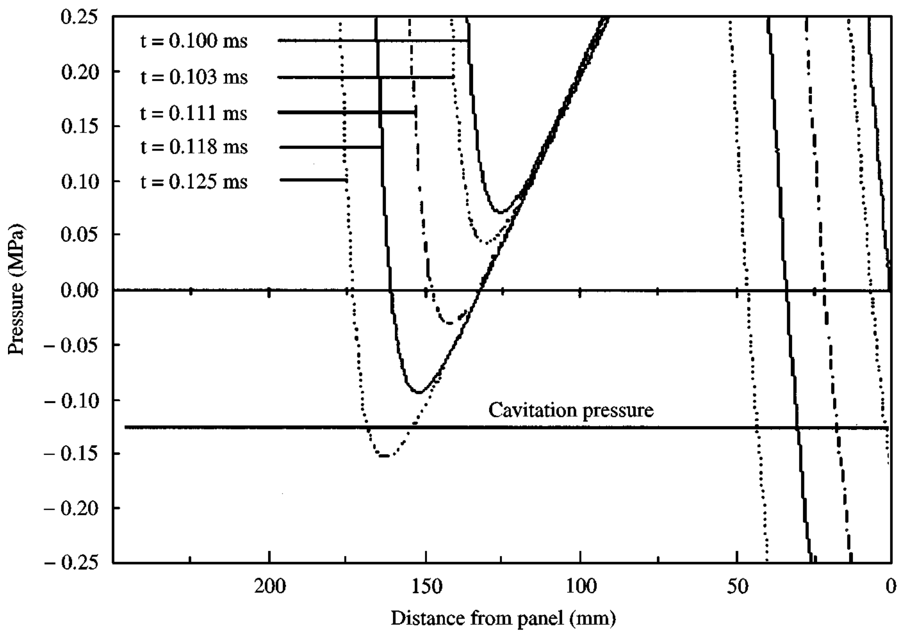

Figure 2. Cavitation in front of a sandwich panel.

4.1. Formulation for Internal Points

For the discrete numerical model we use a mesh with the nodes posed at points (Figure 3) $x_1, = 0$, $x_2 = h_c$, $x_N = d$, $x_{N+1} = d + h_0 \dots x_X = X$, where h_c and h_0 are the distances between the nodes in the core and fluid, respectively, and as it is impossible in numerical methods to deal with infinity region we introduce fictitious boundary $x = X$. The mesh widths are constant, although not in principle for further developments. We suppose that

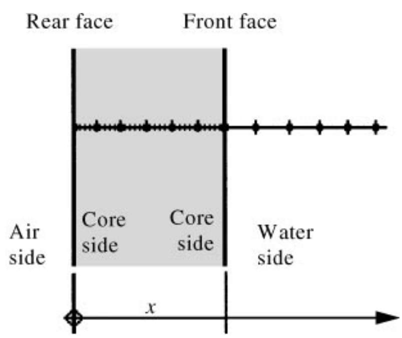

Figure 3. Nodes in numerical model.

pressure and velocity between the nodes x_{j-1}, x_j at the instant $t = m\tau$ (τ is the time step) are constant and denote them as $p_{j-1/2}$, $u_{j-1/2}$. The values of these functions at the instant $t = (m+1)\tau$ will be denoted as $p^{j-1/2}$, $u^{j-1/2}$.

Initial values of $p_{j-1/2}$, $u_{j-1/2}$ ($t = 0$) are defined by equation (4). The calculations of $p^{j-1/2}$, $u^{j-1/2}$ are performed in two steps.

In the first step, we calculate some auxiliary values U_j, P_j in accordance with the following formulas:

$$U_j = \frac{u_{j-1/2} + u_{j+1/2}}{2} - \frac{p_{j-1/2} + p_{j+1/2}}{2\rho c},$$
$$P_j = \frac{p_{j-1/2} + p_{j+1/2}}{2} - \rho c\,\frac{u_{j-1/2} + u_{j+1/2}}{2}. \quad (14)$$

It is important to notice that equation (14) is the analytical solution of the so-called problem of "decay of acoustical gaps".

Hence, if we consider the solution of the problem with piecewise constant initial data $p_{j-1/2}$, $u_{j-1/2}$ at segment $x_{j-1} < x < x_j$ and $p_{j+1/2}$, $u_{j+1/2}$ at segment $x_j < x < x_{j+1}$, see Figure 4, then the solution of the governing equation (5) will be

In zone I: $x_{j-1} + ct < x < x_j - ct$, $\quad p = p_{j-1/2}$, $u = u_{j-1/2}$.

In zone II: $x_j + ct < x < x_{j+1} - ct$, $\quad p = p_{j+1/2}$, $u = u_{j+1/2}$.

In zone III: $$\begin{cases} U_j = \dfrac{u_{j-1/2} + u_{j+1/2}}{2} - \dfrac{p_{j-1/2} + p_{j+1/2}}{2\rho c}, \\ P_j = \dfrac{p_{j-1/2} + p_{j+1/2}}{2} - \rho c\,\dfrac{u_{j-1/2} + u_{j+1/2}}{2}. \end{cases}$$

The system of waves indicated in Figure 4, keeps its pattern until the time instant when sound waves from two nearby nodes meets. The procedure for construction of the analytical solution, tracing for all acts of such meetings, is very complicated and, in the second step, approximate values of $p^{j-1/2}$, $u^{j-1/2}$ are defined on the basis of the laws of conservation. In the second step of the method we integrate equation (5) and obtain

$$\oint_\Gamma \rho u\,\mathrm{d}x - \oint_\Gamma p\,\mathrm{d}t = 0,$$
$$\oint_\Gamma \frac{p}{c^2}\,\mathrm{d}x - \oint_\Gamma \rho u\,\mathrm{d}t = 0, \quad (15)$$

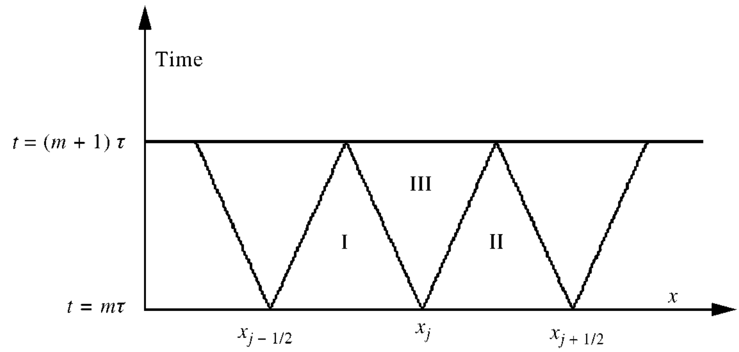

Figure 4. Piecewise constant data.

where Γ may be an arbitrarily closed contour at the plane (x, t). The first of equations (15) is the law of conservation of momentum ($\int \rho u \, dx$ is momentum and $\int p \, dt$ is pressure impulse), the second is the law of conservation of mass as in acoustics, and $p = c^2(\rho - \rho_0)$. We now apply the first relation to a rectangular cell Γ, delimited by the lines $x = x_{j-1}$, $x = x_j$ and $t = m\tau$, $t = (m + 1)\tau$, and we obtain,

$$\int_{x_{j-1}}^{x_j} u(x, (m + 1)\tau \, dx = \int_{x_{j-1}}^{x_j} u(x, m\tau) \, dx - \frac{1}{\rho_0} \int_{m\tau}^{(m+1)\tau} [p(x_j, t) - p(x_{j-1}, t)] \, dt. \quad (16)$$

We assume function $u(x, t)$ to be constant in the segment $[x_{j-1}, x_j]$ and to be equal to $u_{j-1/2}$. At the left and right sides of Γ, $p(x, t)$ is equal to P_{j-1} and P_j, and as a result we get from equation (16)

$$\frac{1}{h} \int_{x_{j-1}}^{x_j} u(x, (m + 1)\tau \, dx = u_{j-1/2} - \frac{\tau}{h} \frac{1}{\rho_0} (P_j - P_{j-1}). \quad (17)$$

To the left in this equation is the averaged value of velocity at the instant $t = (m + 1)\tau$, and it is this value we assume to be a value for the velocity at this instant. Treating the second of equations (15) in a similar way, we finally have the formulas for the second step of the numerical method:

$$\begin{aligned} u^{j-1/2} &= u_{j-1/2} - \frac{\tau}{h} \frac{1}{\rho} (P_j - P_{j-1}), \\ p^{j-1/2} &= p_{j-1/2} - \frac{\tau}{h} \rho c^2 (U_j - U_{j-1}). \end{aligned} \quad (18)$$

4.2. Formulas for Boundary Points

Let us assume that at the point $x = x_1$ we have boundary conditions

$$\alpha(t) u(0, t) + \beta(t) p(0, t) = f(t).$$

Godunov (1978) suggests that the values of P_0 and U_0 should be calculated from the system

$$\begin{aligned} \tilde{\alpha} U_0 + \tilde{\beta} P_0 &= \tilde{f}, \\ U_0 - \frac{1}{\rho_0 c_0} P_0 &= u_{1/2} - \frac{1}{\rho_0 c_0} p_{1/2}. \end{aligned} \quad (19)$$

Here $\tilde{\alpha}, \tilde{\beta}, \tilde{f}$ are some approximations of α, β, f during the time interval $(t, t+\tau)$. From the physical point of view, equations (19) describe the problem of "decay of boundary acoustical gap". For the problem in question the coefficients α, β, f are not known beforehand but have to be calculated together with the hydrodynamical values. We further denote $w(m\tau)$ as w^m. Replacing equations (1) and (3) by their finite-difference analogues and joining them with the second of equations (19), we then consider the rear and front face of the sandwich as two different boundaries.

Starting with the rear face (point $x_1 = 0$) and considering the core to be an acoustical medium with density ρ_c and sound velocity c_c, the equations of acoustics stay true for internal points ($0 < x < d$, $n = 1, 2, \ldots, N$) and their finite-difference analogues. Now, we obtain the following system of algebraic equations over w^{m+1}, P_0, U_0:

$$
\begin{aligned}
&Kw^m + \frac{m_f}{\tau^2}\left(w^{m+1} - 2w^m + w^{m-1}\right) = P_0,\\
&\frac{w^{m+1} - w^{m-1}}{2\tau} = -U_0,\\
&U_0 - \frac{1}{\rho_c c_c} P_0 = u_{1/2} - \frac{1}{\rho_c c_c}\, p_{1/2}.
\end{aligned} \tag{20}
$$

At the front face (point $x_N = d$) and the region ($x > d$, $n = N+1, N+2, \ldots, X$), i.e., the fluid, the equations are valid and we have five equations:

$$
\begin{aligned}
&\frac{m_f}{\tau^2}\left(w^{m+1} - 2w^m + w^{m-1}\right) = P_{\mathrm{N}} - P_{CN},\\
&\frac{w^{m+1} - w^{m-1}}{2\tau} = -U_N,\\
&U_N = U_{CN},\\
&U_N - \frac{1}{\rho_0 c_0}\, P_N = u_{N+1/2} - \frac{1}{\rho_0 c_0}\, p_{N+1/2},\\
&U_{CN} + \frac{1}{\rho_c c_c} P_{CN} = u_{N-1/2} + \frac{1}{\rho_c c_c} p_{N-1/2}.
\end{aligned} \tag{21}
$$

The solution of this linear system of algebraic equations gives the values of w^{m+1}, P_{CN}, P_N, U_{CN}, U_N. These values are to be used in the determination of the new pressures and velocities.

As to the fictitious boundary point $x_X = X$, necessary values of P_X, U_X may be prescribed in any arbitrary way, only if the signal from boundary point does not have time to reach the point in question. In particular, it is possible to assume that $U_X = P_X = 0$.

4.3. Numerical Cavitation Model

The condition of the appearance of cavitation is formulated as

$$
p + p_h \le p_c, \tag{22}
$$

where p_h is hydrostatic pressure and set at 0·10 MPa and p_c is the cavitation pressure. The pressure in zones with cavitation changes quite weakly and we approximately assume it equal to pressure in water vapour $p_v = 1000$ Pa. Therefore, the equation of motion in zones with cavitation is

$$\frac{\mathrm{d}u}{\mathrm{d}t} = 0.$$

This means that water particles in zones with cavitation move without acceleration and interaction with each other.

4.4. Initial Values of Panel Displacement

As an approximation for the initial deformation of the front face of the sandwich we use the equations derived by Hayman. Starting with equation (6) that gives the motion of the front face of the sandwich and substituting in the expressions for the pressures, equations (4), (9) and (10), we obtain

$$p_0 \mathrm{e}^{-\alpha t} + B_1 \mathrm{e}^{-\alpha t} + B_2 \mathrm{e}^{-\beta t} - A(\mathrm{e}^{-\alpha t} - \mathrm{e}^{-\beta t}) = m_f \, \ddot{w}_f(t).$$

Integrating this equation twice and keeping in mind the initial conditions, equations (2), we obtain an expression for the initial deformation of the front face:

$$w(t) = \left[(p_0 + B_1 - A) \frac{\mathrm{e}^{-\alpha t}}{\alpha^2} + (B_2 + A) \frac{\mathrm{e}^{-\beta t}}{\beta^2} \right] \Big/ m_f. \tag{23}$$

5. NUMERICAL EXAMPLE

5.1. Example by Hayman

To verify the numerical model, the same input as in the analytical solution by Hayman, was used in this example and the solution obtained is given in Figure 5.

Figure 5 should be compared with Figure 2 and will be seen that the agreement between the numerical and analytical solutions is very good, concerning the pressure in the water in front of the sandwich panel.

The numerical calculations for the pressure were made in accordance with the following procedure: (a) initial pressure and velocity are given by equations (2); (b) initial deformation of the front face is calculated from equation (23); (c) the auxiliary values U_j, P_j are calculated according to equation (14); (d) P_0, U_0, w^{m+1} are calculated by solving equations (20) and w^{m+1}, P_{CN}, P_N, U_{CN}, U_N by equations (21), and we set $P_X = U_X = 0$; and (e) $p^{j+1/2}$, $u^{j+1/2}$ are calculated according to equation (18) and if condition (22) is fulfilled, we set $p^{j+1/2} = p_v$, $u^{j+1/2} = u_{j+1/2}$.

It is necessary to notice here that the values of steps h and τ must not only satisfy the condition of stability of finite-difference scheme ($\tau/h \leq 1/c_0$), but also have a certain physical meaning, namely that they characterize space and time scales of an "elementary" cavity. Zones with cavitation with smaller dimensions cannot exist.

5.2. Transverse Response of Sandwich Panel

A simple 1-D finite-element model (Figure 6), in the transverse direction of the sandwich panel, consisting of 4 bar elements for each sandwich face and 6 bar elements for the core, is used in the analysis.

The sandwich panel is assumed to consist of a 90 mm thick PVC core and 7 mm thick GRP faces. The properties of the core are $E_{\mathrm{comp}} = 400$ MPa, $E_{\mathrm{tens}} = 300$ MPa, $\rho_c =$

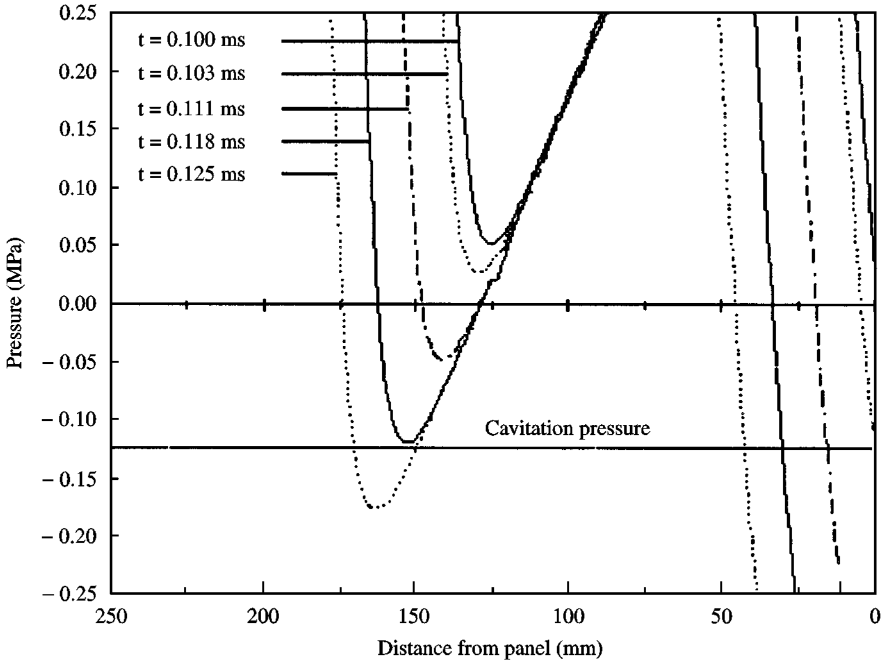

Figure 5. Numerical solution of example by Hayman (1995).

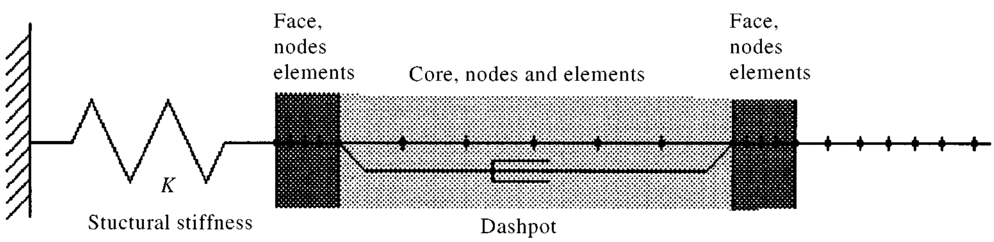

Figure 6. Finite-element model.

250 kg/m^3 and for the faces $E_{comp} = 3000$ MPa, $E_{tens} = 30$ MPa, $\rho_f = 2100$ kg/m^3. The structural stiffness (K) is assumed to be $1{\cdot}00 \times 10^7$ N/m and the damping (C) in the core is set at 1000 Ns/m. For the loading, of the sandwich panel, we assume a 200 kg explosive HBX charge at a distance of 20 m from the structure.

The numerical solution to this problem was obtained by solving the fluid/structural coupled problem. For the structure, equation (1) was used and the applied load was simultaneously solved for, as described in Section 5.1. From the solution of this coupled system we can for example extract the pressure on the structural surface, as in Figure 7, and also the cavitation zones in the fluid and the transverse structural displacement.

Figure 7 shows the pressure on the panel surface and the free-field pressure as a function of time during the analysis. First we have the pressure doubling, followed by a very rapid decay of the pressure. Just before cavitation occurs at the panel surface, we get a pressure increase. Then follows a cavitation period and a first "pressure pulse". After that comes a cavitation period and another "pressure pulse", this time weaker. This sequence of

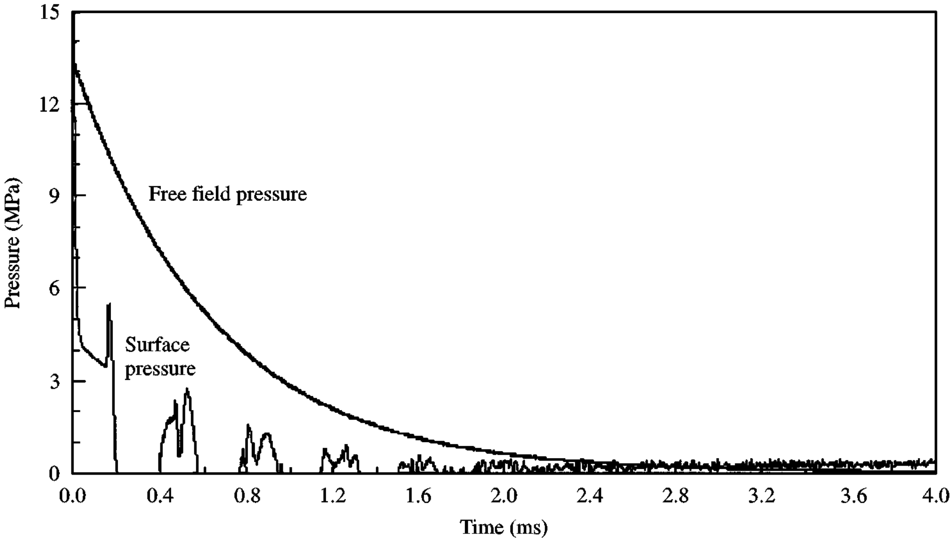

Figure 7. Pressure on panel surface.

cavitation and pressure pulse is then repeated a number of times until the pressure slowly starts to increase again. When this slow increase begins, the formation of cavitation zones adjacent to the panel surface have stopped. The free-field pressure in the figure, is the calculated pressure in the fluid as if no structure were present, equation (4).

This behaviour of the pressure, on the structural surface, is quite different from what is expected on a homogeneous panel, where we also have a quick decay, followed by a longer cavitation period and then the slowly increasing pressure with time.

5.2.1. *Cavitation zones in the fluid*

Since the numerical model consists of nodes in the fluid, it is possible to trace the cavitation zones. As can be seen from Figures 8(a) and 8(b), cavitation starts about 0·180 ms from the time when the shock wave arrives at the structure and is developed approximately 180 mm away from the structural surface. At time 0·200 ms, cavitation begins to develop at the fluid/structure interface and the pressure drops to zero, in accordance with Figure 7. Comparing the pressure curve in Figure 7 and the cavitation zones in Figure 8(a) and 8(b) it can be seen that when there is a layer of fluid attached to the structure, there is also a pressure on the structure.

After approximately 1·60 ms, the pressure on the structural surface slowly starts to increase and the cavitation zone grows in size and moves away from the structure.

5.2.2. *Transverse structural response*

In Figure 9(a), the displacements for the first 1·60 ms are plotted and we can see that after approximately 0·085 ms the loading starts to affect the rear face. During the loading phase the front face, water side and core side, are deformed more or less equally. When cavitation starts, the loading is taken away and the displacement rate of the front face slows down. At this time the displacement of the rear face has accelerated and the deformation exceeds that of the front face. Now, the whole section is in tension and, due to the much lower tension

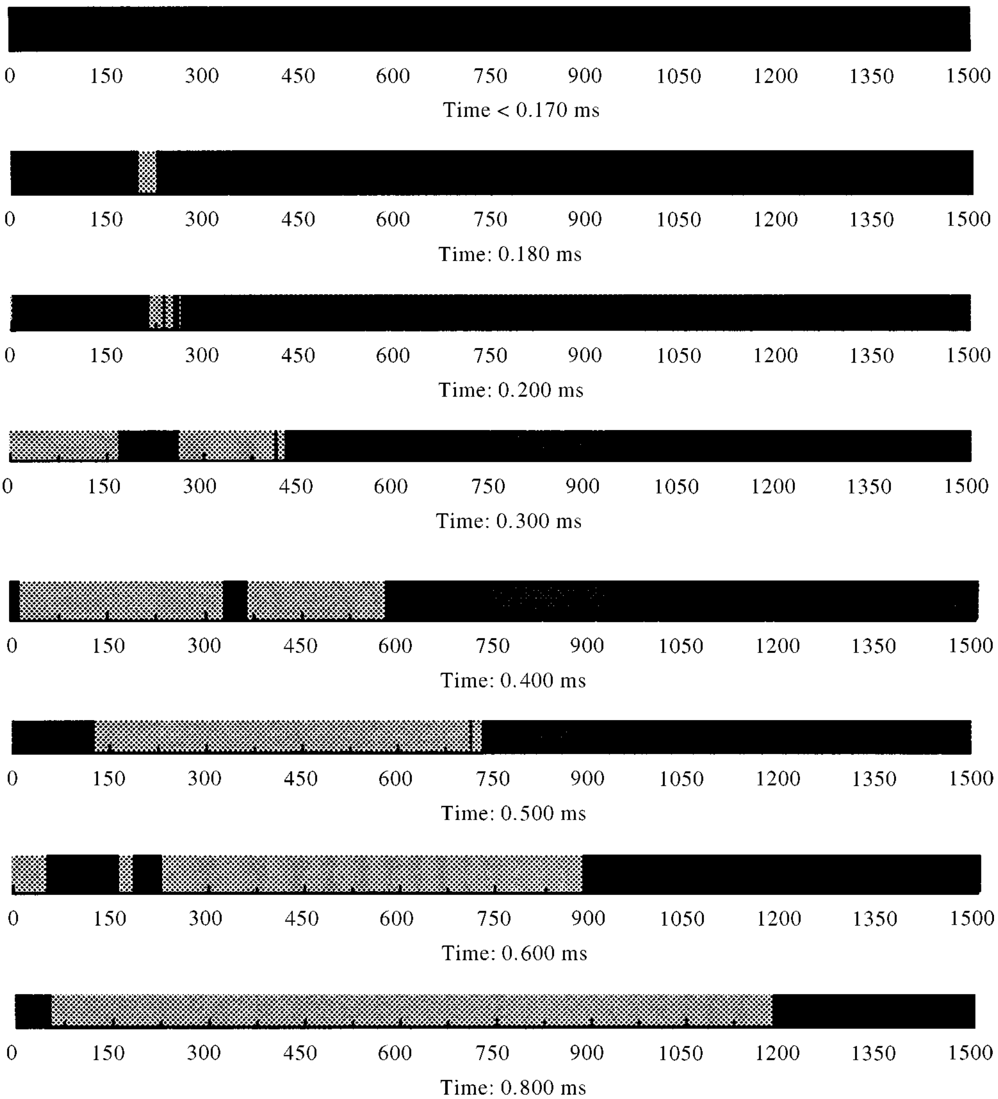

Figure 8(a). Cavitation zones (in mm) at distinct times in front of the sandwich structure.

properties of the faces, each side of the front and rear face is not deformed equally. Shortly after, the front face catches up the rear face, and a similar cycle starts again with one compression phase followed by one tension phase. From Figures 7 and 9 it can be seen that the compression phases of the core approximately coincide with the "pressure pulses" in the loading on the panel surface. As the pressure becomes smaller, the oscillation of the faces and core decays and finally stops.

Figure 9(b) is an amplification of the first 0·50 ms of the displacement of the sandwich. From this figure we can see the difference in displacement of the front and rear face of the sandwich. Further, the difference on the core side and fluid side is visible for the front face and core side and the air side for the rear face. During the first 0·23 ms the whole section is in compression, but during the time interval of 0·23–0·43 ms the section is in tension and

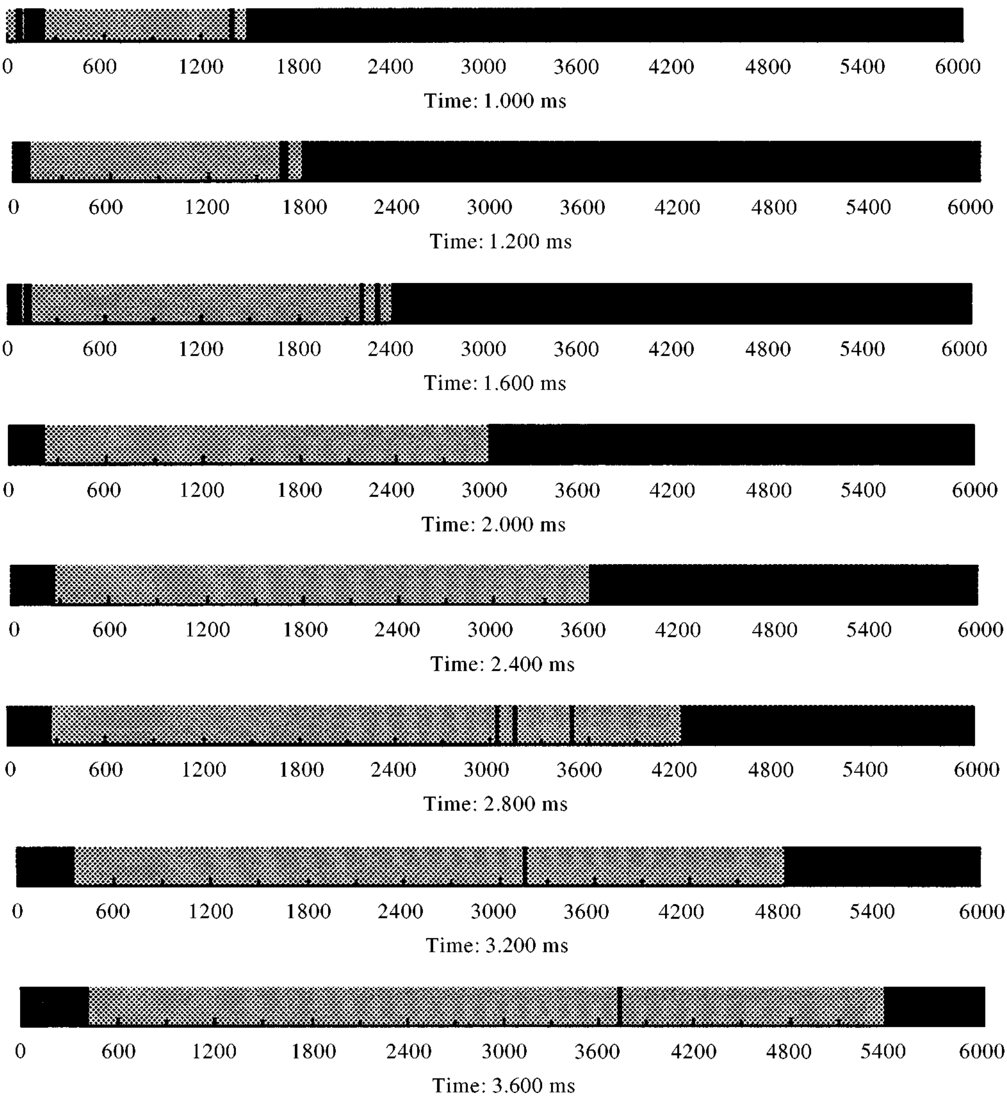

Figure 8(b). cavitation zones (in mm) at distinct times in front of the sandwich structure.

the much poorer tension properties in the faces cause significant strain in the faces, as can be seen in Figure 10. The tension modulus is only 1% of the compression modulus. The decaying oscillation of the faces depends mainly on the fact that the load is decaying in magnitude and pulsation and slowly converging to an almost constant value. In this case the faces are subjected to strain in the range of 2·5–3·5%. The question is whether this is enough to cause delamination or not. In order to answer this question we need to make a detailed analysis of the laminate in the transverse direction or obtain reliable results from delamination tests.

The core has a tension modulus that is 75% of the compression modulus, and the variation in compression and tension strain is not that significant. The decaying amplitude is due to the damping properties of the core. In this case we have a maximum strain of about 1% and for most types of PVC cores used in sandwich construction this is not critical.

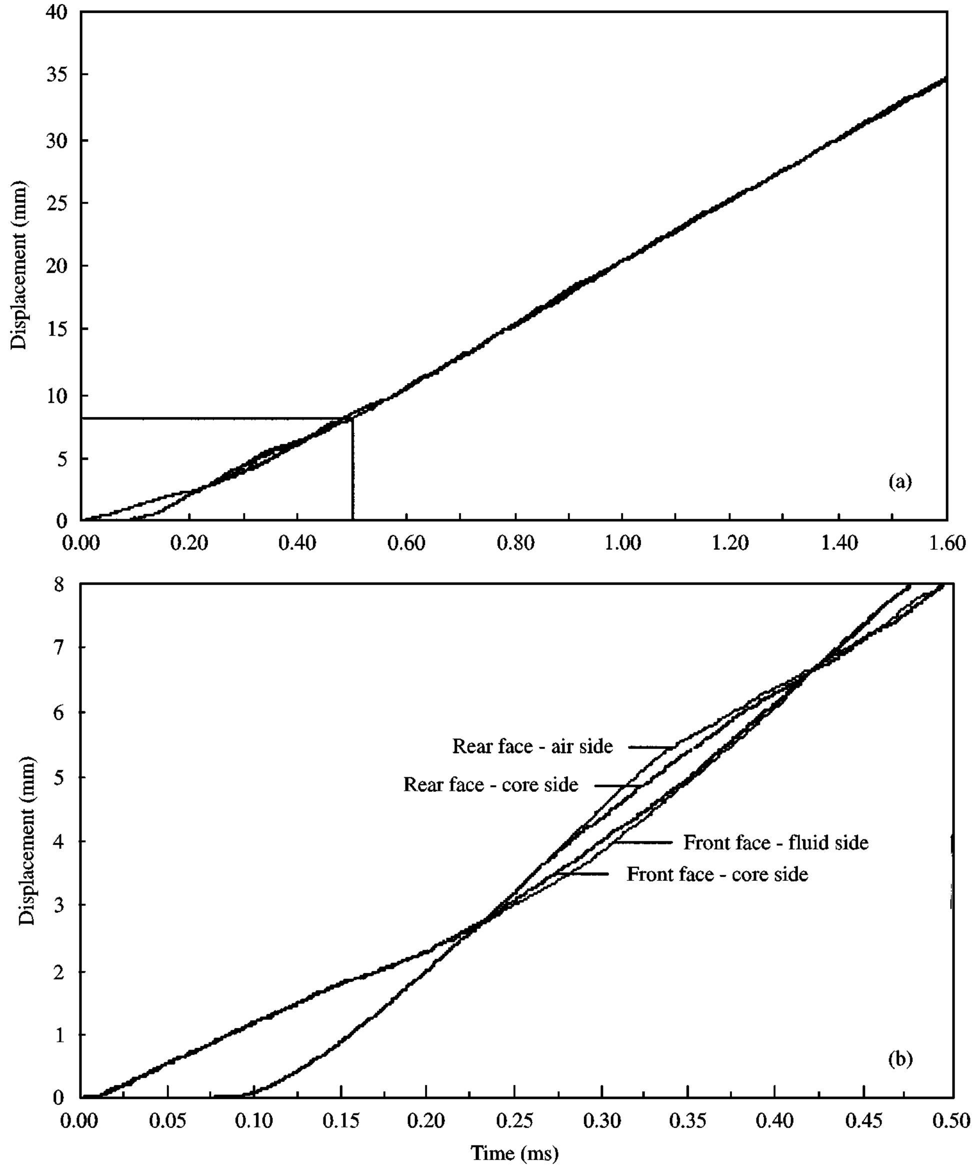

Figure 9(a). Transverse displacement of the sandwich; (b) transverse displacement of the sandwich, first 0.50 ms.

6. CONCLUSION

The numerical method developed in this paper shows good agreement with the analytical solution by Hayman. In one example, the pressure in the fluid in front of the panel was compared to Hayman's, and the agreement between the analytical and numerical solution, is very good.

The fluid–structure interaction between a fluid and a sandwich shell structure is quite different from that of the interaction between a fluid and a homogeneous shell structure. In the case of a sandwich structure, initially two cavitation zones open up, while for

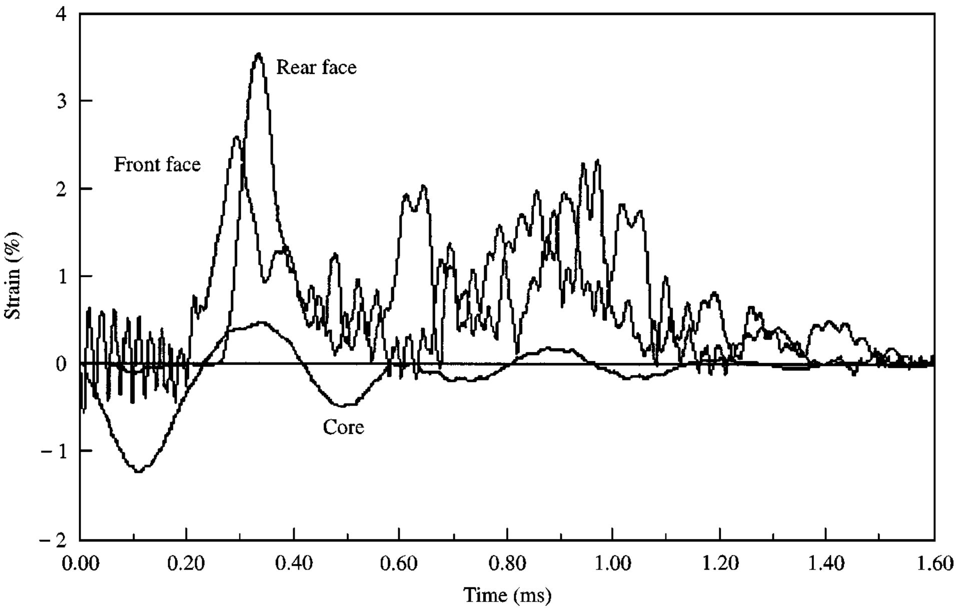

Figure 10. Transverse core and face strains.

a homogenous structure only one occurs. The distance from the structure at which these open is quite different for the two cases. The numerical method also allows the tracing of cavitation zones, in the fluid, in front of the sandwich structure, and it was found that when there is a pressure on the structural surface there is also an amount of fluid attached to the structure.

By using a simplified 1-D finite element model, of a sandwich section, in the transverse direction, the nonlinear response has been investigated. The results show that there is a strong interaction between the displacement of the sandwich and the pressure on the front face. As the whole sandwich section is moving, the faces oscillate about the core and this oscillation may give rise to significant strain in the different parts of the sandwich. The poorer tension properties of the faces, in the transverse direction, give rise to strains that might lead to delamination of the faces when they are in tension. The strain in the core is more or less equal in compression and tension during the oscillation, due to its properties, and damping causes the oscillation to decay.

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